Spectral Methods in Matlab

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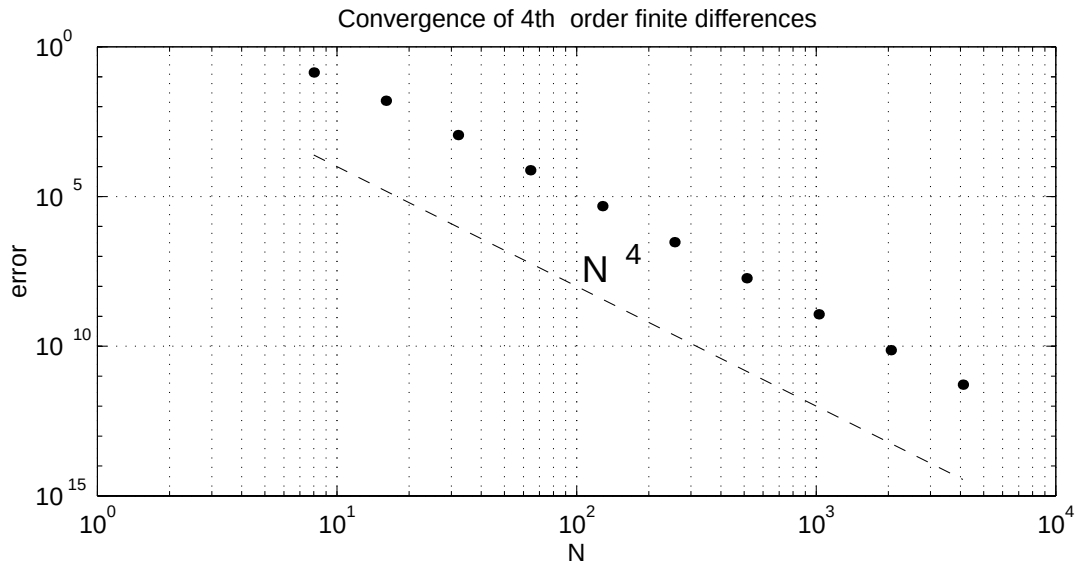
Contents

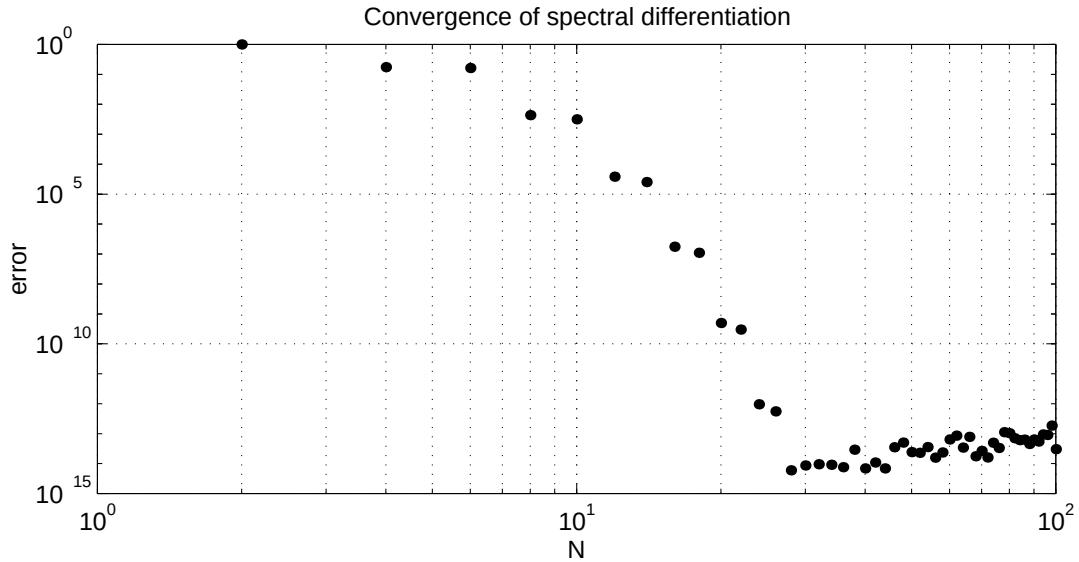
Foreword

A Note on the Matlab Programs

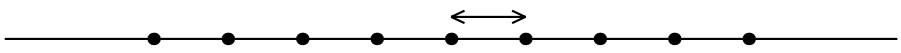
Acknowledgments

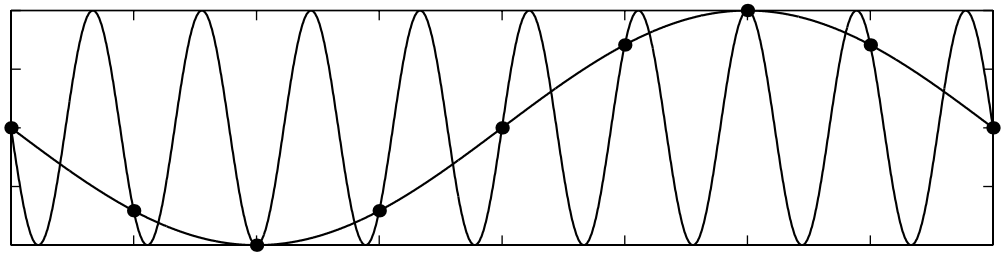
1. Differentiation Matrices

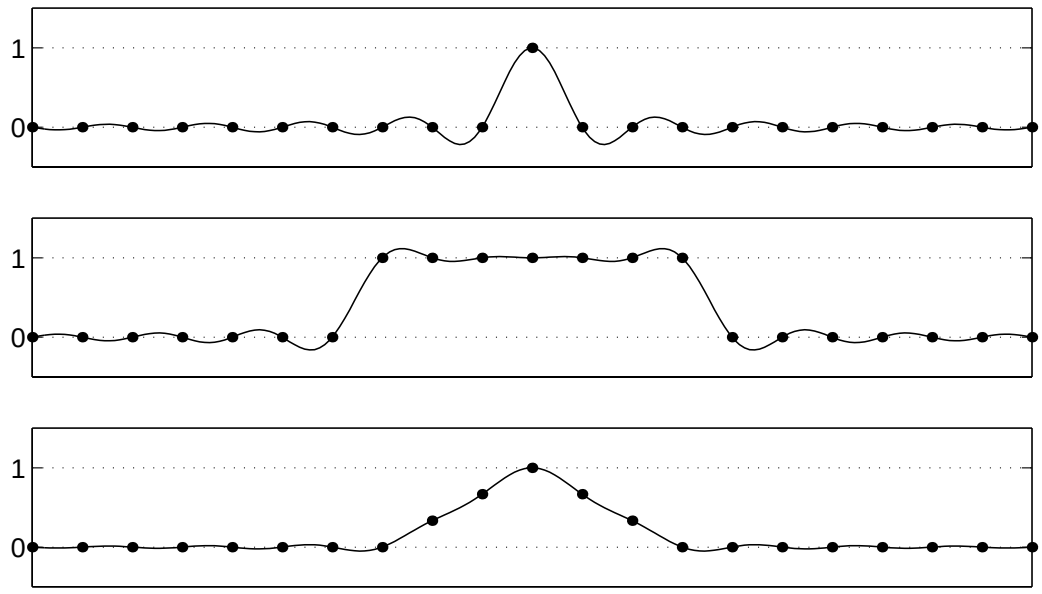

Convergence of 4th order finite differences
error
N
N 4
10^0
10^5
10^10
10^15
10^0
10^1
10^2
10^3
10^4


Convergence of spectral differentiation
error
N
10^0
10^5
10^10
10^15
10^0
10^1
10^2

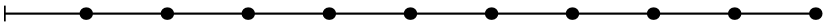
2. Unbounded Grids: the Semidiscrete Fourier Transform

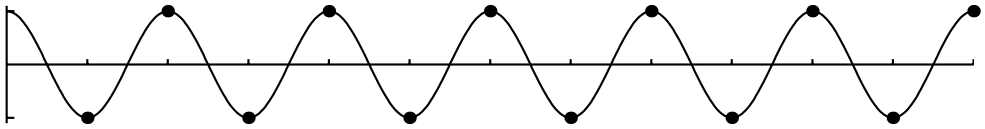


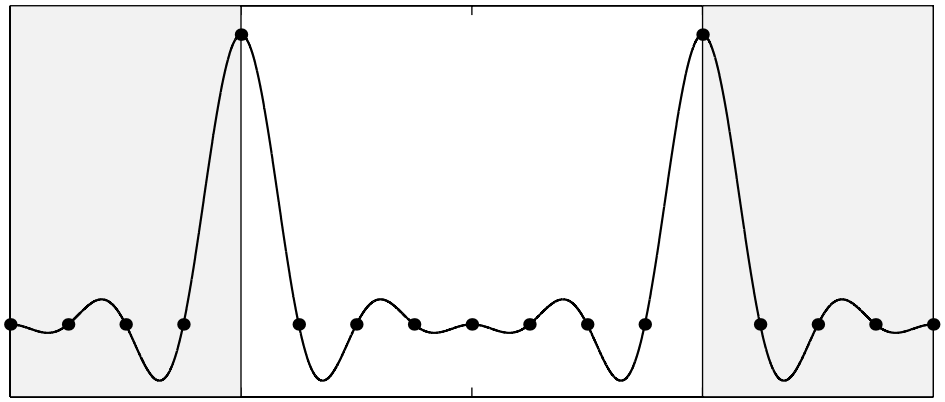


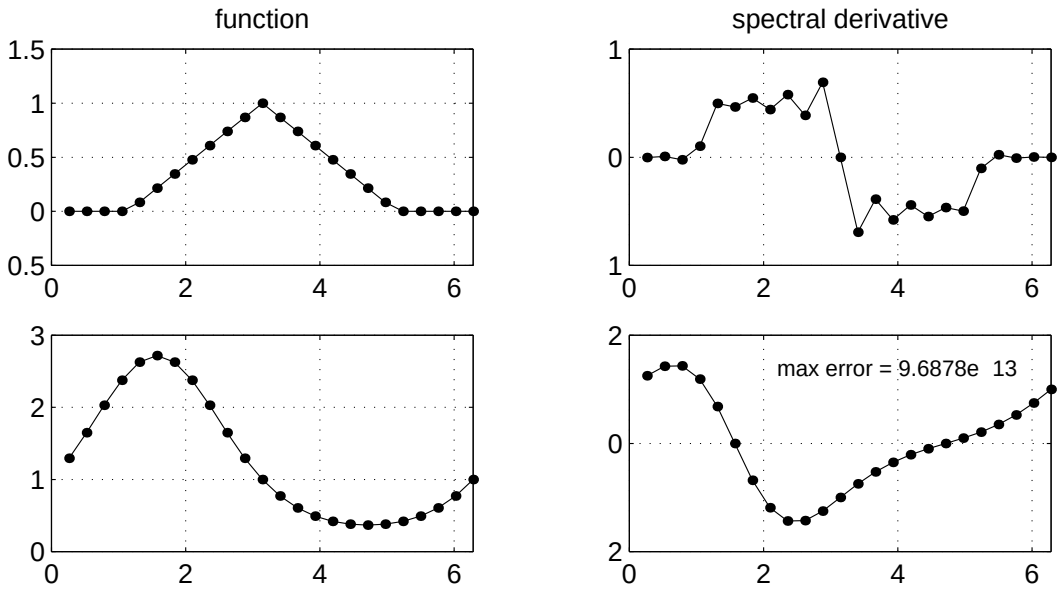

1
0
1
0
1
0

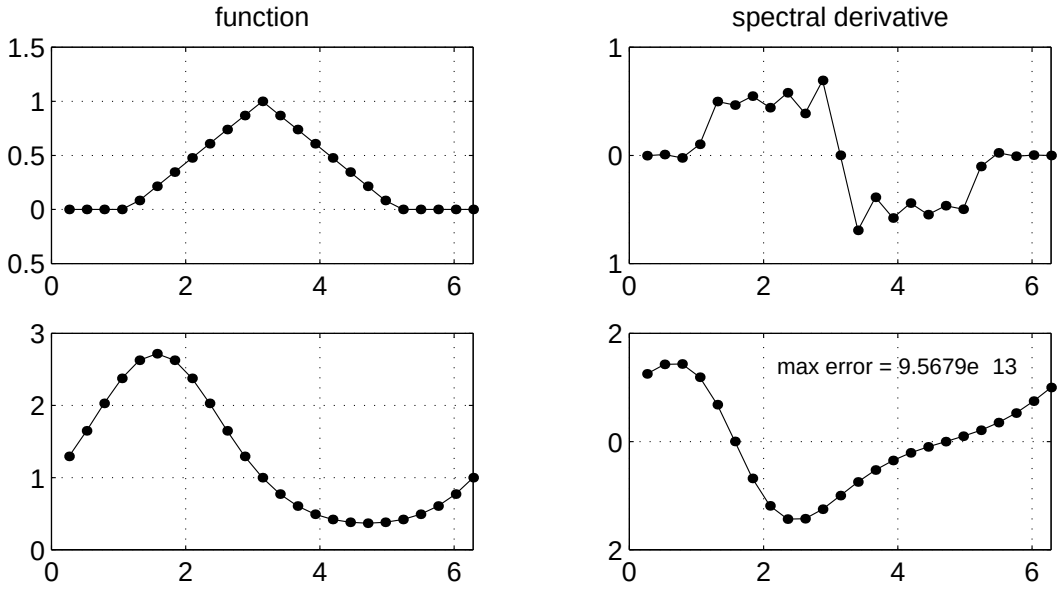
3. Periodic Grids: the DFT and FFT

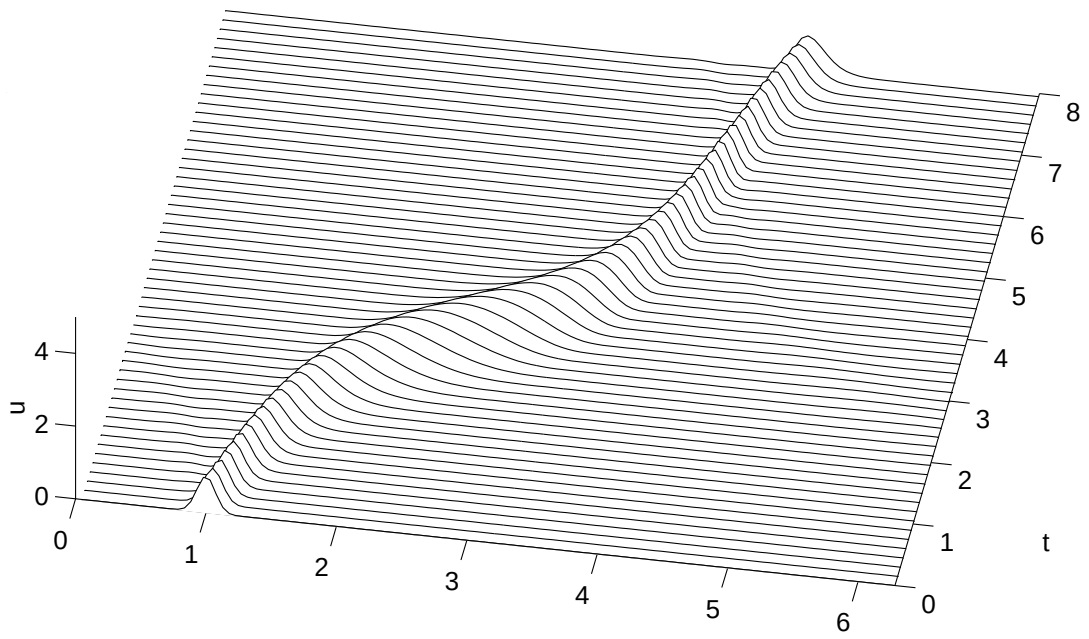




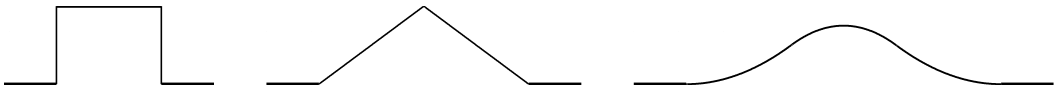


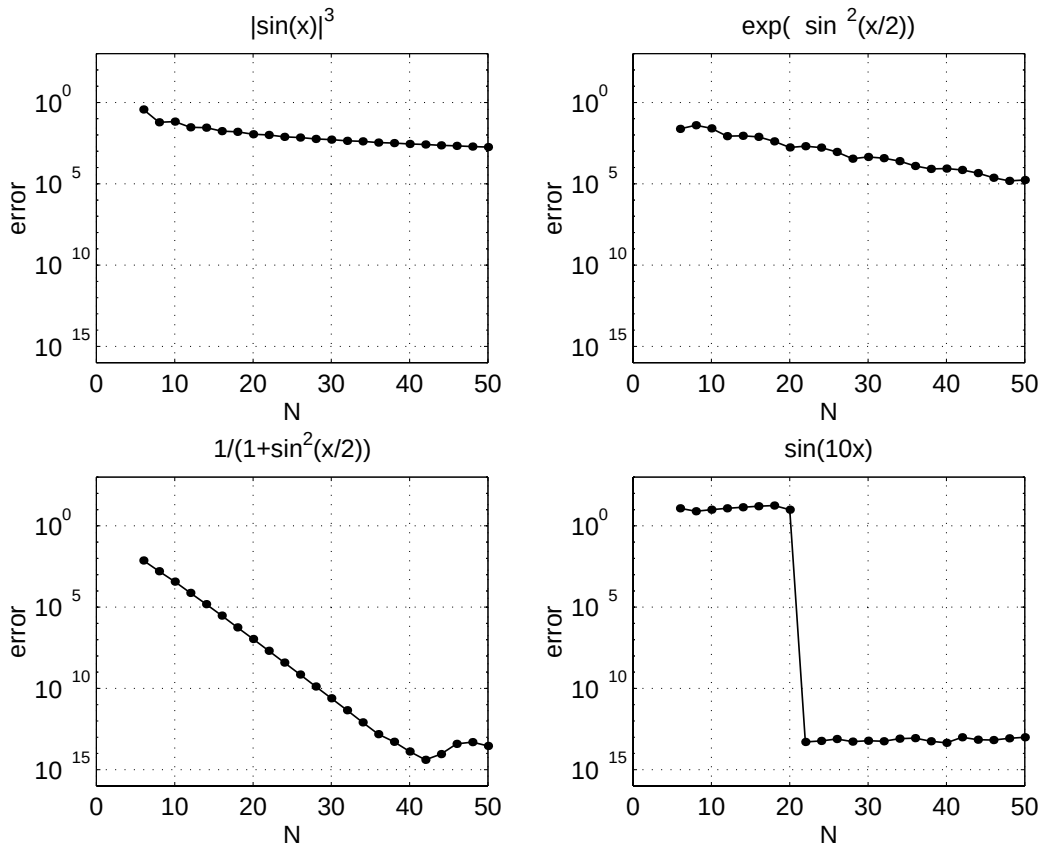

function
spectral derivative
max error = 9.6878e 13


function
spectral derivative
max error = 9.5679e 13

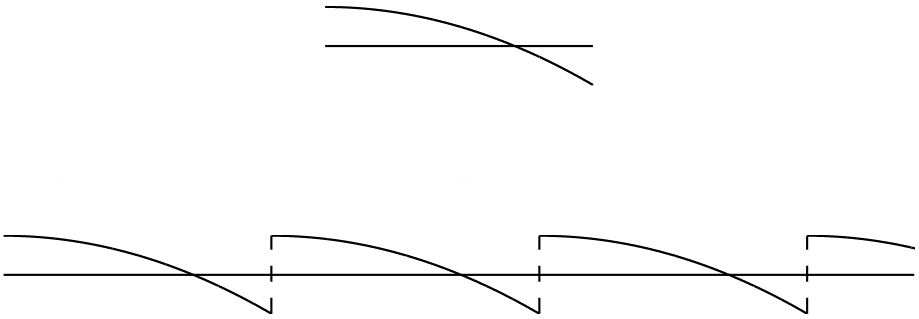

u
0
2
4
0
1
2
3
4
5
6
t
0
1
2
3
4
5
6
7
8

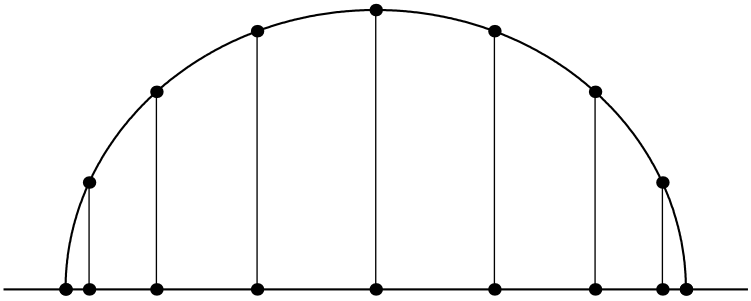
4. Smoothness and Spectral Accuracy

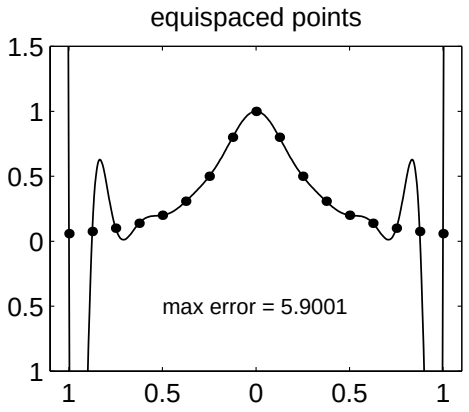


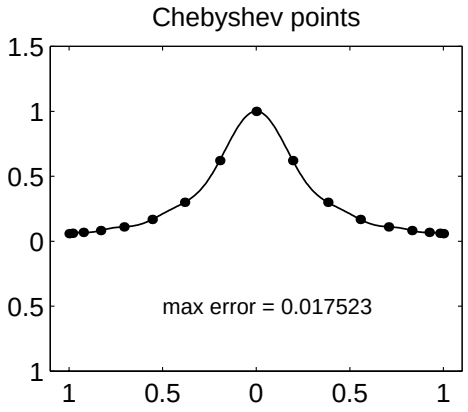

|sin(x)|^3
exp(sin 2(x/2))
1/(1+sin^2(x/2))
sin(10x)
error
N
10^0
10^5
10^10
10^15
0
10
20
30
40
50

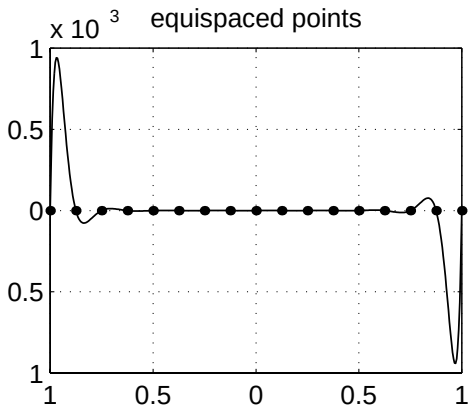
5. Polynomial Interpolation and Clustered Grids

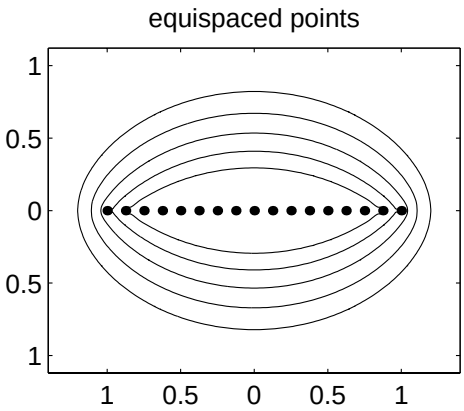


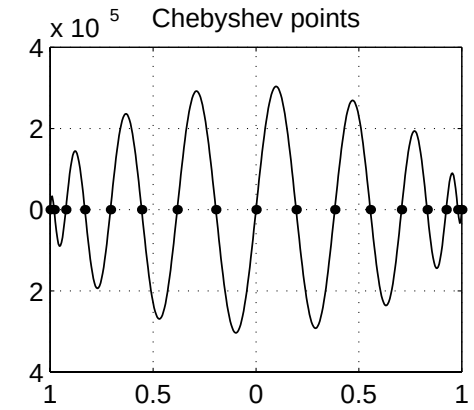


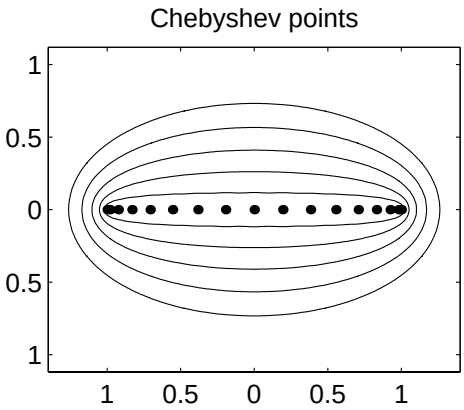

equispaced points
1.5
1
0.5
0
0.5
1
max error = 5.9001
1
0.5
0
0.5
1


Chebyshev points
1.5
1
0.5
0
0.5
1
max error = 0.017523
1
0.5
0
0.5
1

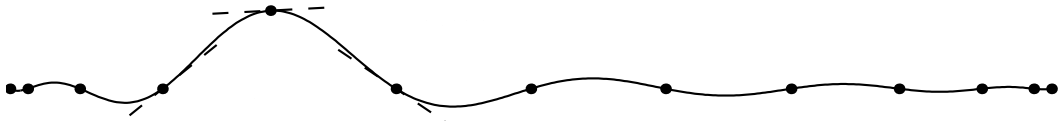

x 10 3
equispaced points
1
0.5
0
0.5
1
1
0.5
0
0.5
1

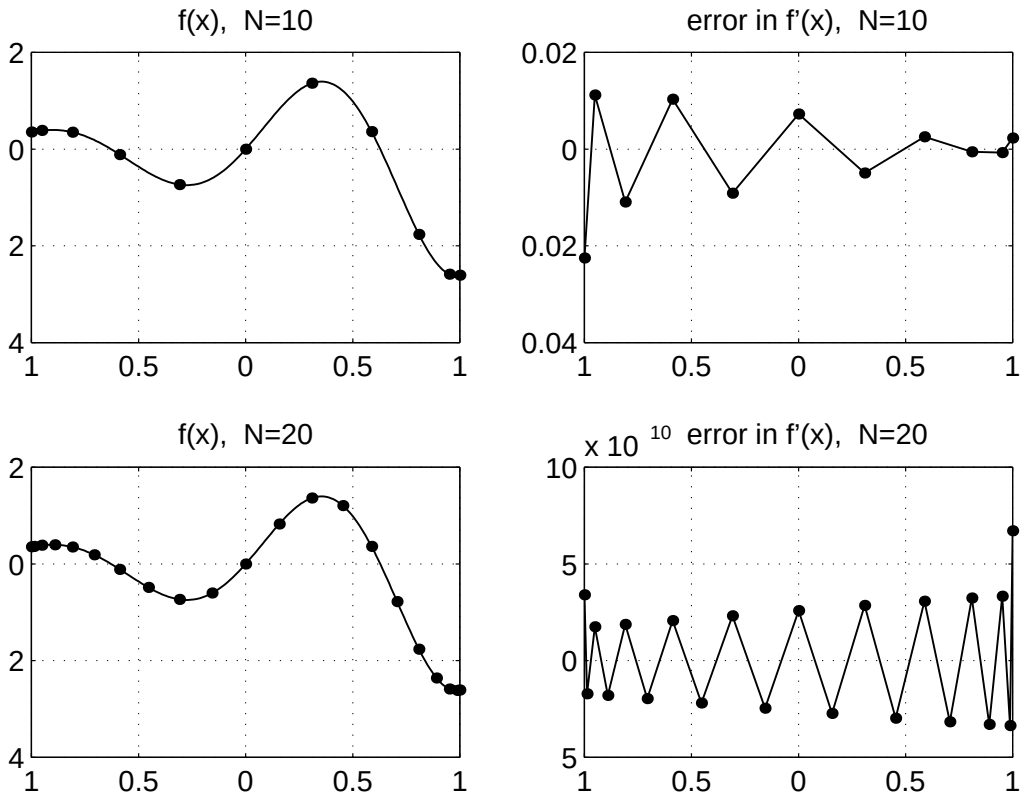

equispaced points
1
0.5
0
0.5
1
1
0.5
0
0.5
1

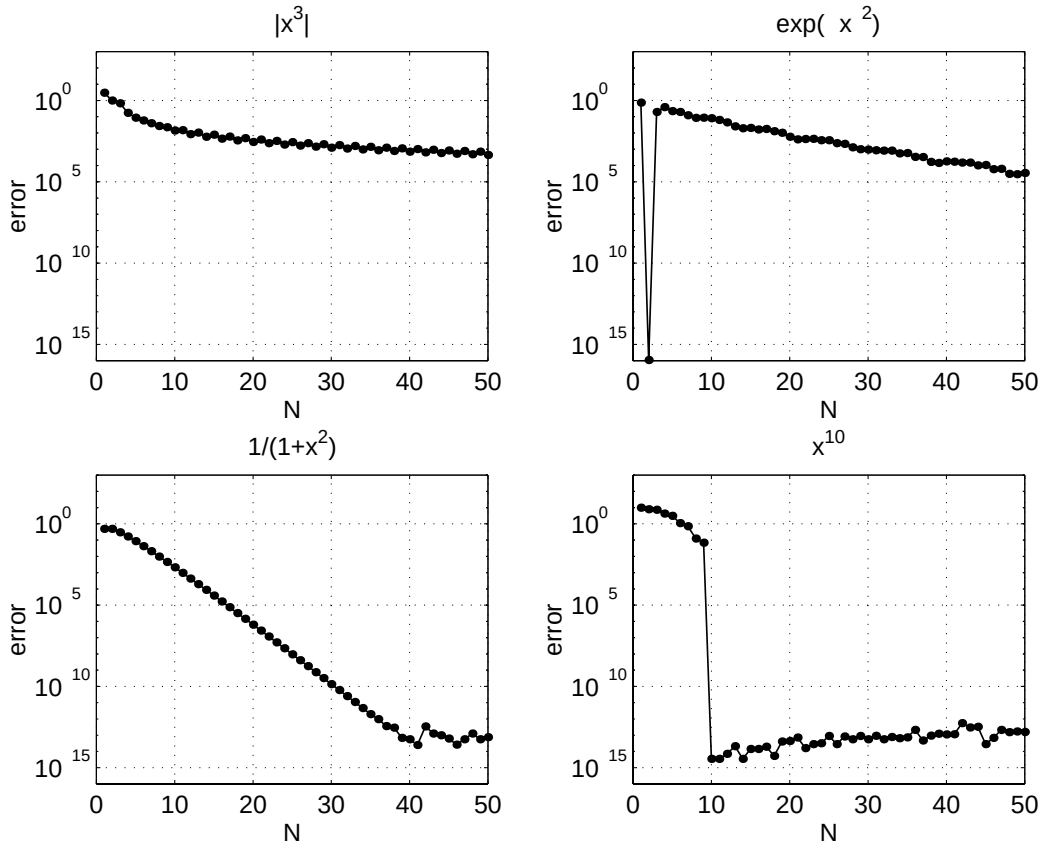

x 10 5
Chebyshev points
4
2
0
2
4
1
0.5
0
0.5
1


Chebyshev points
1
0.5
0
0.5
1
1
0.5
0
0.5
1

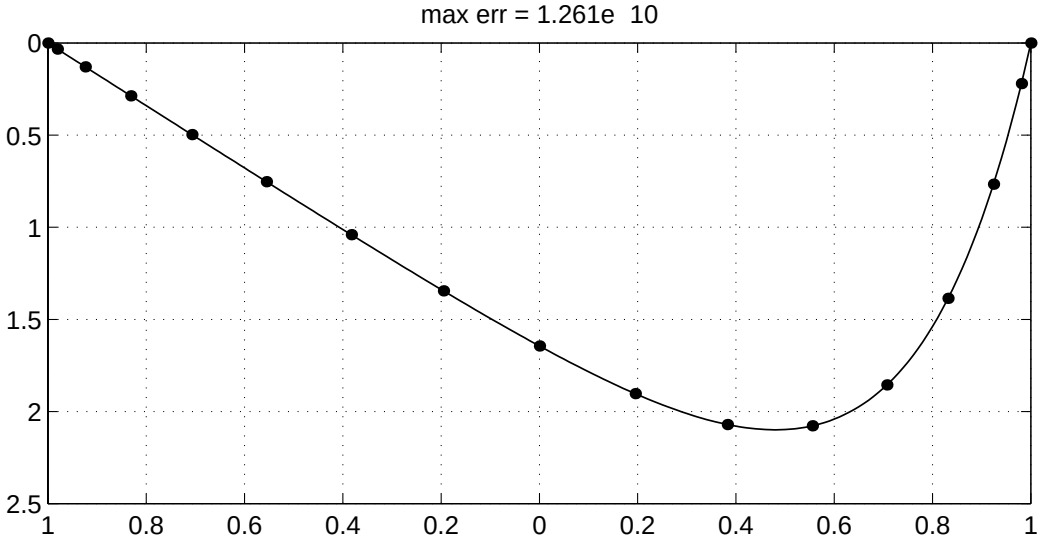
6. Chebyshev Differentiation Matrices

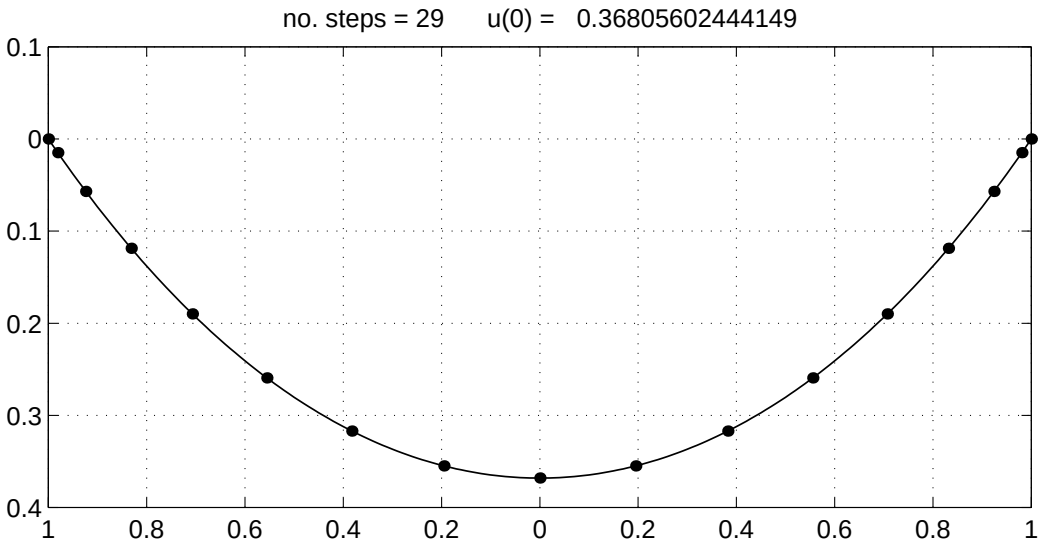


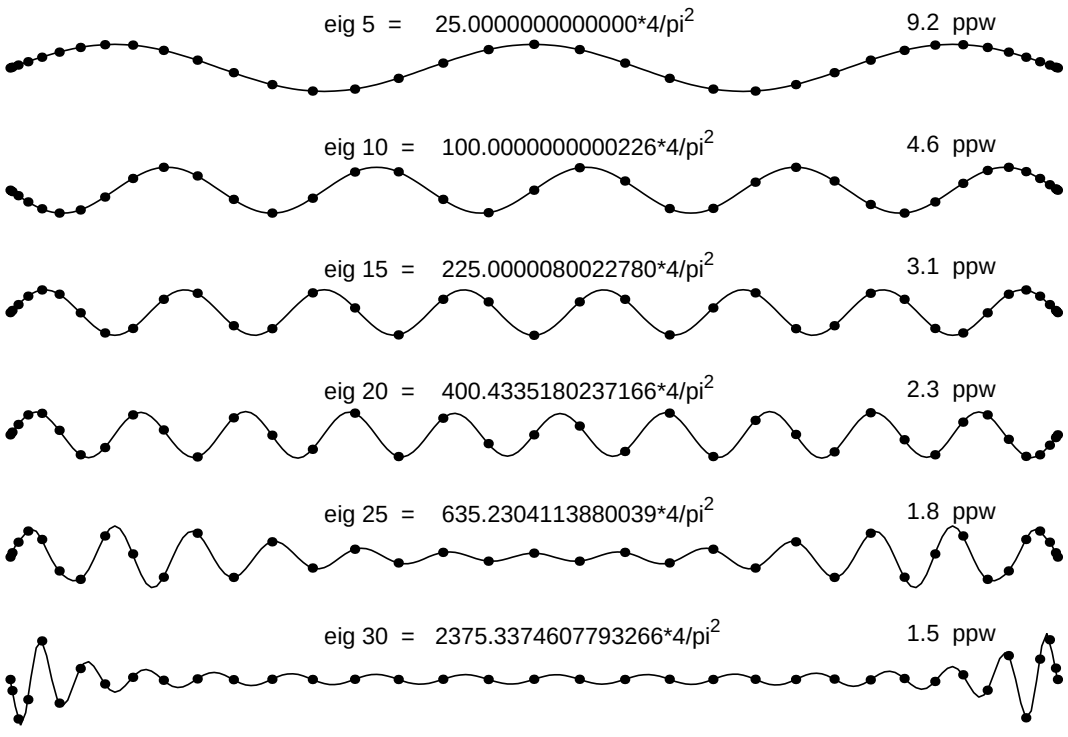

f(x), N=10
error in f'(x), N=10
f(x), N=20
x 10 10 error in f'(x), N=20

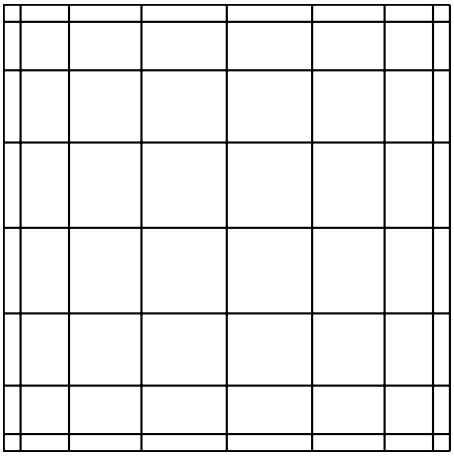

|x^3|
exp(x 2)
1/(1+x^2)
x^10
error
N
10^0
10^5
10^10
10^15
0
10
20
30
40
50

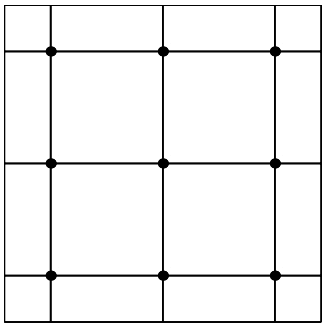
7. Boundary Value Problems

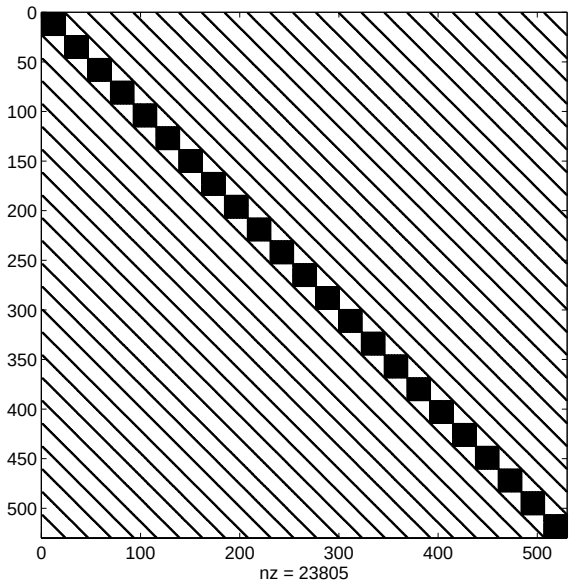

max err = 1.261e 10
0
0.5
1
1.5
2
2.5
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1

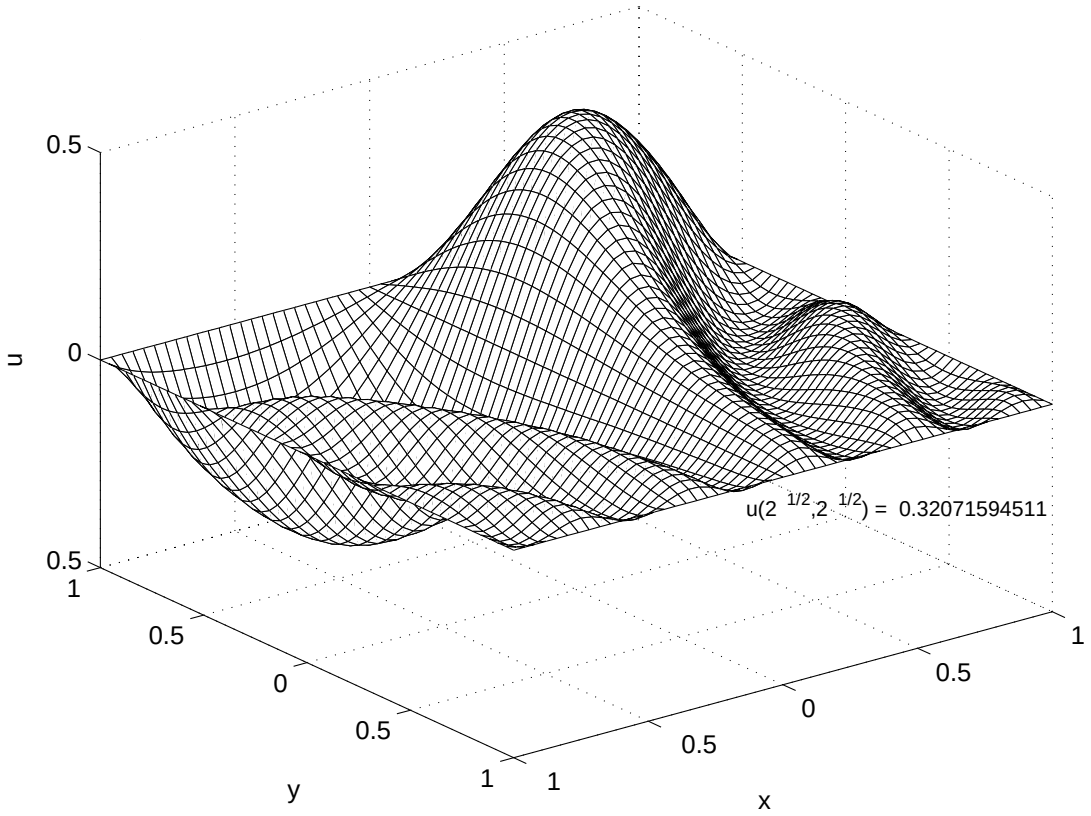

no. steps = 29 u(0) = 0.36805602444149
0.1
0
0.1
0.2
0.3
0.4
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1

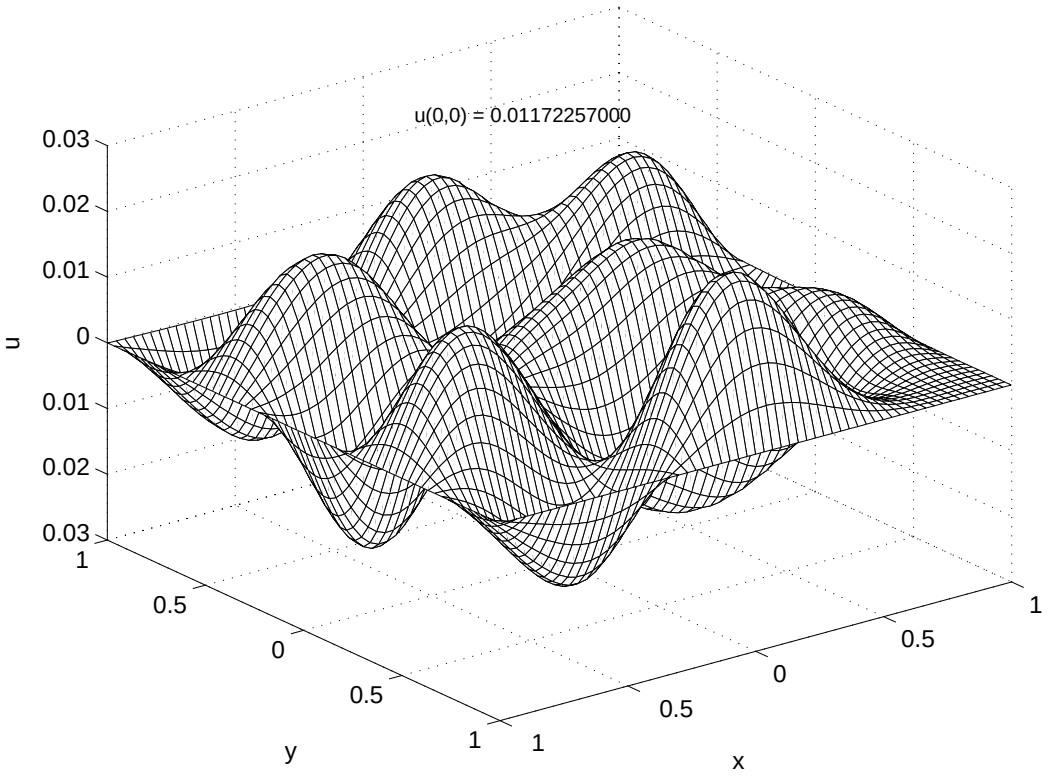

eig 5 = 25.0000000000000*4/pi2
9.2 ppw
eig 10 = 100.0000000000226*4/pi2
4.6 ppw
eig 15 = 225.0000080022780*4/pi2
3.1 ppw
eig 20 = 400.4335180237166*4/pi2
2.3 ppw
eig 25 = 635.2304113880039*4/pi2
1.8 ppw
eig 30 = 2375.3374607793266*4/pi2
1.5 ppw

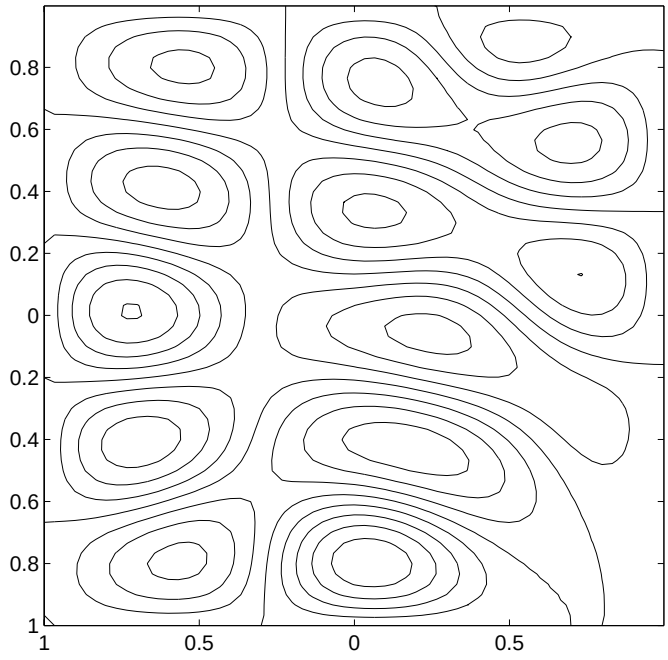




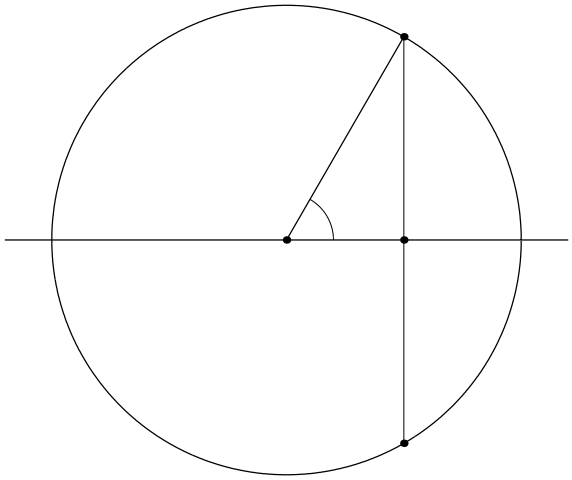

0
50
100
150
200
250
300
350
400
450
500
0
100
200
300
400
500
nz = 23805

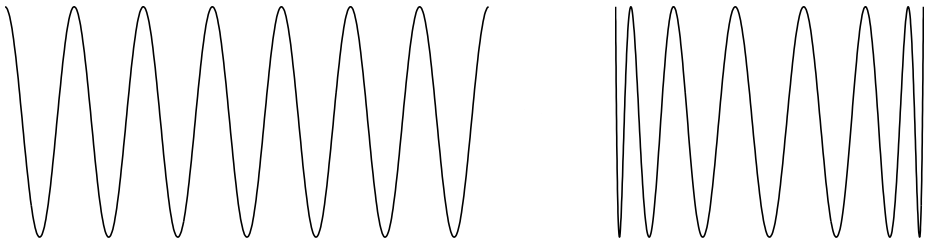

0.5
0
0.5
u
1
0.5
0
0.5
1
y
1
0.5
0
0.5
1
x
u(2 1/2,2 1/2) = 0.32071594511

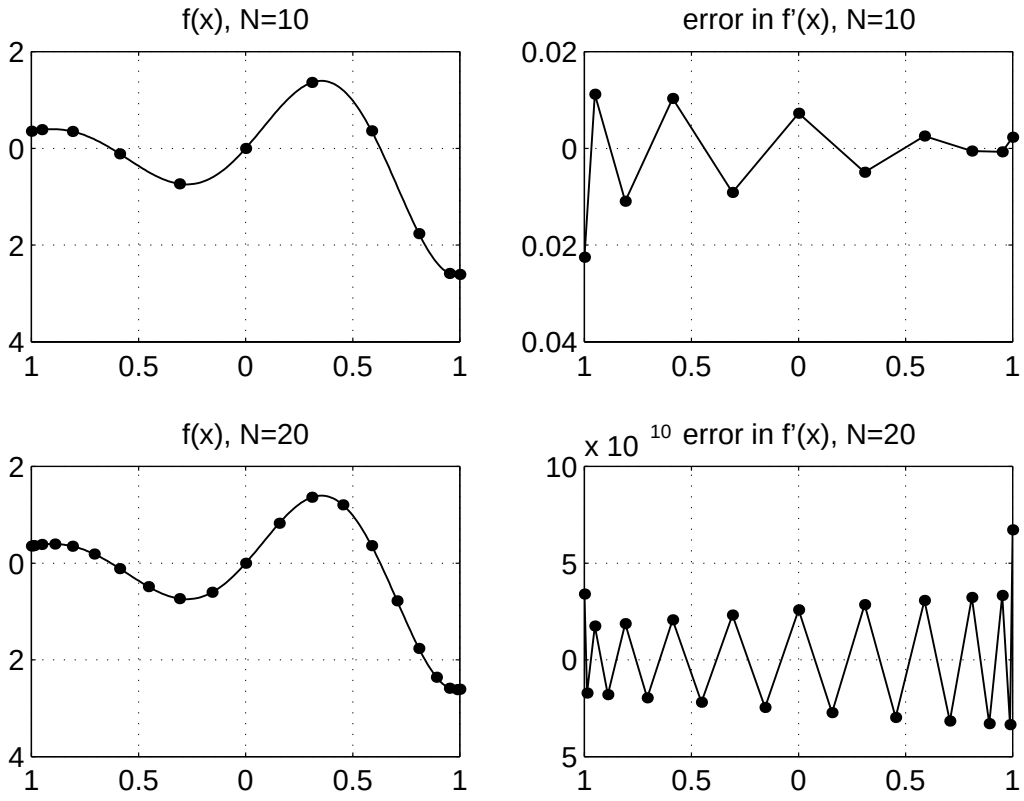

u(0,0) = 0.01172257000
0.03
0.02
0.01
0
0.01
0.02
0.03
u
1
0.5
0
0.5
1
y
1
0.5
0
0.5
1
x

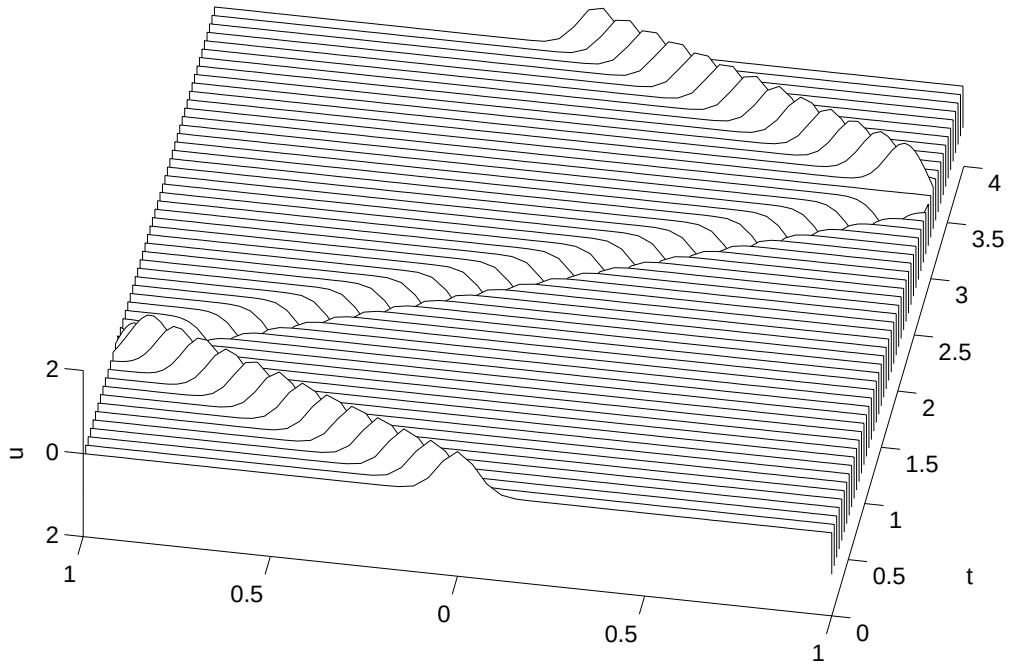

0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
1
0.5
0
0.5

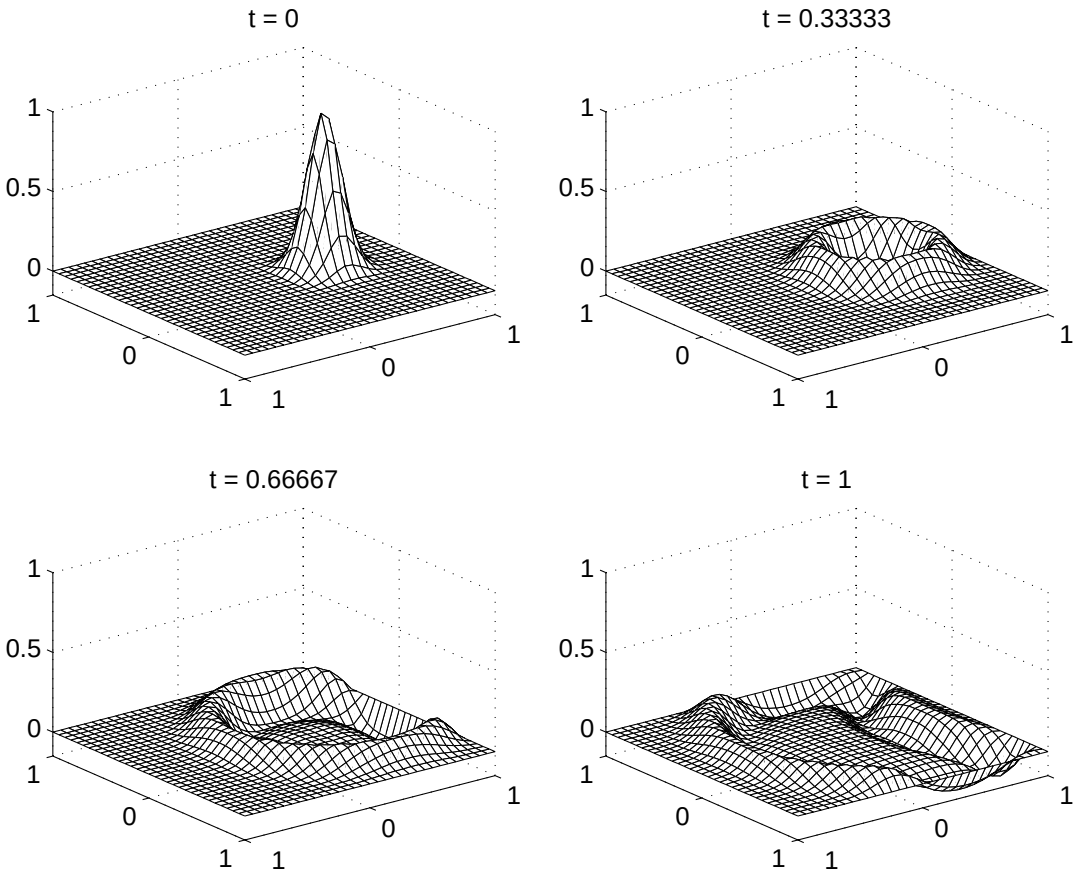
8. Chebyshev Series and the FFT

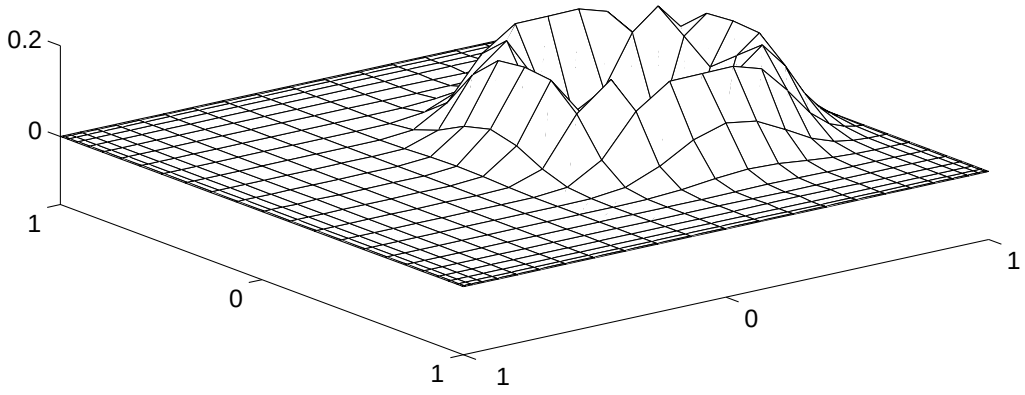





f(x), N=10
error in f'(x), N=10
f(x), N=20
x 10 10 error in f'(x), N=20
2
0
2
4
1
0.5
0
0.5
1
0.02
0
0.02
0.04
10
5
0
5


u
2
0
2
1
0.5
0
0.5
1
0
0.5
1
1.5
2
2.5
3
3.5
4
t


t = 0
t = 0.33333
t = 0.66667
t = 1
1
0.5
0
1
0
1
1
0
1


0.2
0
1
0
1
1
0
1

9. Eigenvalues and Pseudospectra

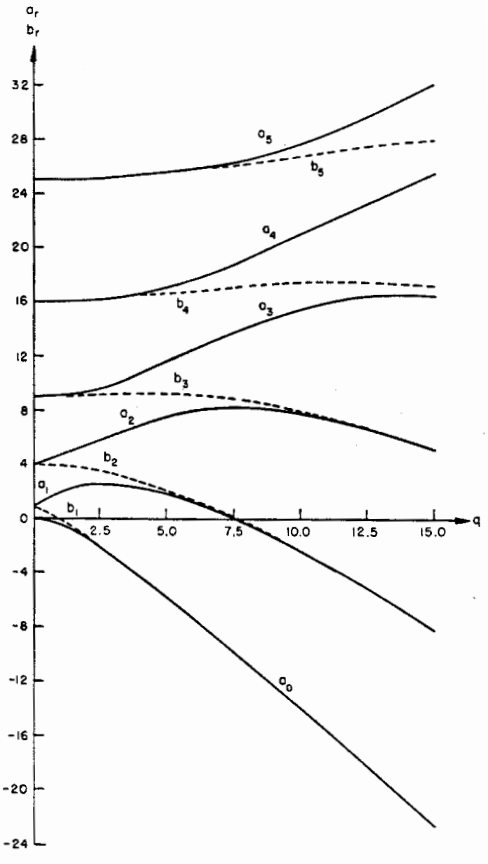
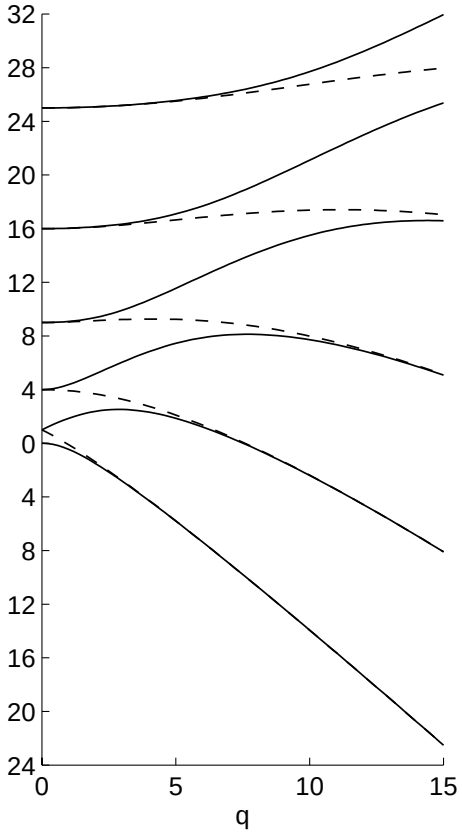
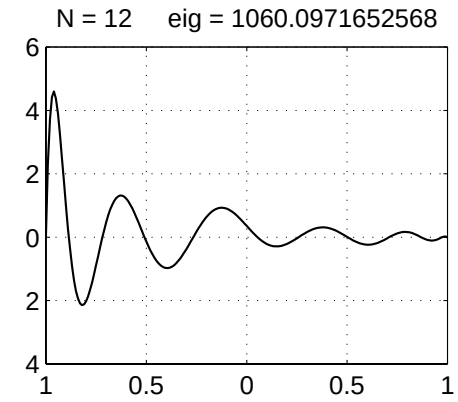
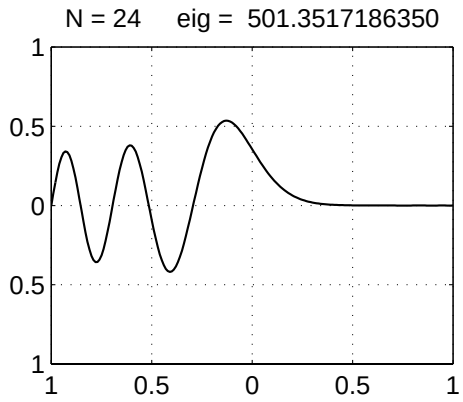
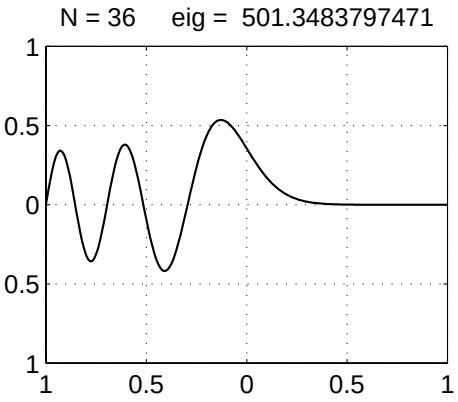


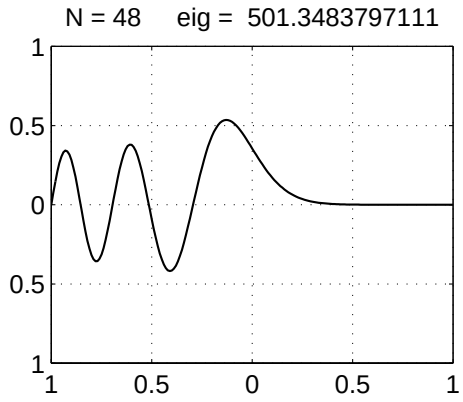
FIGURE 20.1. *Characteristic Values* a_r, b_r $r=0,1(1)5$

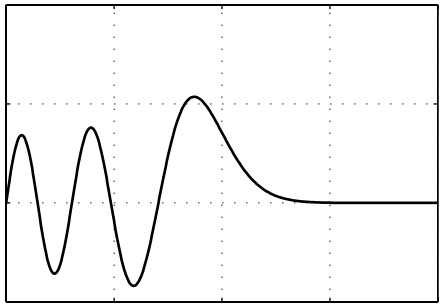


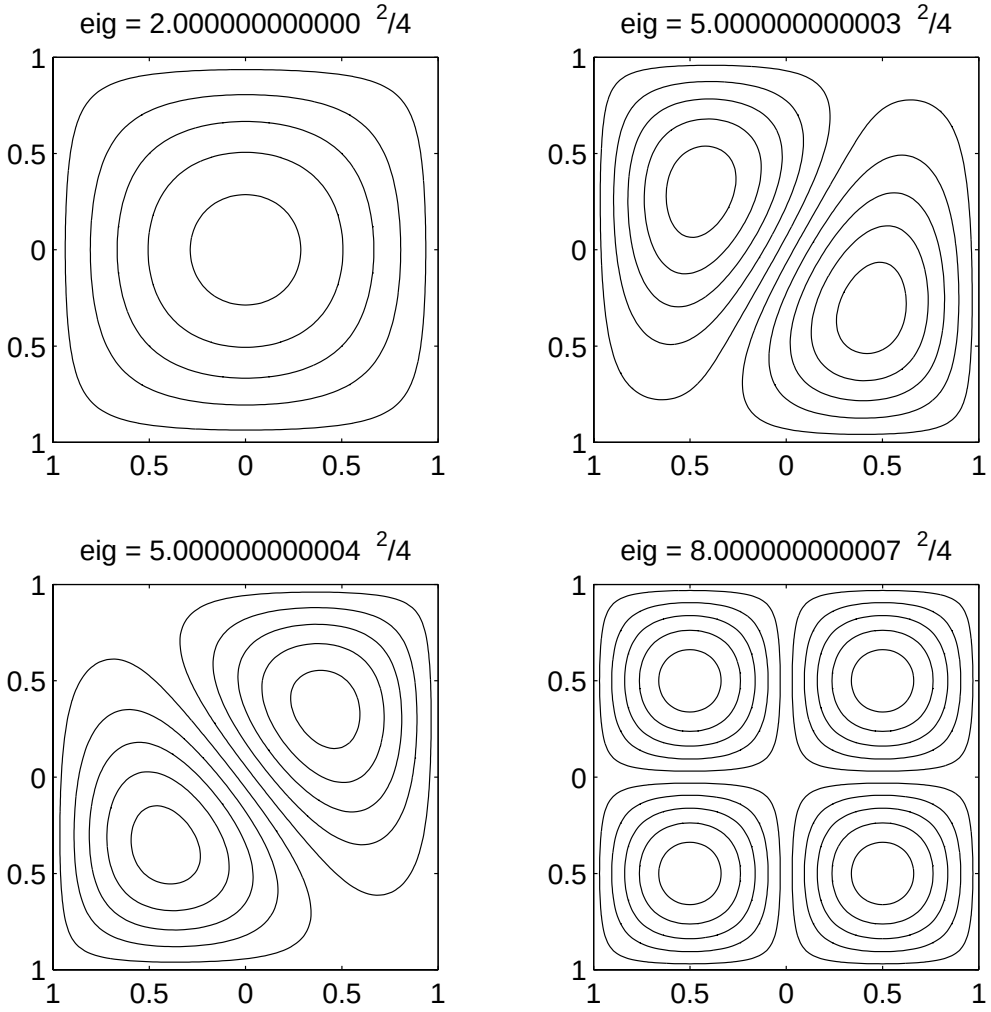

N = 12 eig = 1060.0971652568
6
4
2
0
2
4
1
0.5
0
0.5
1

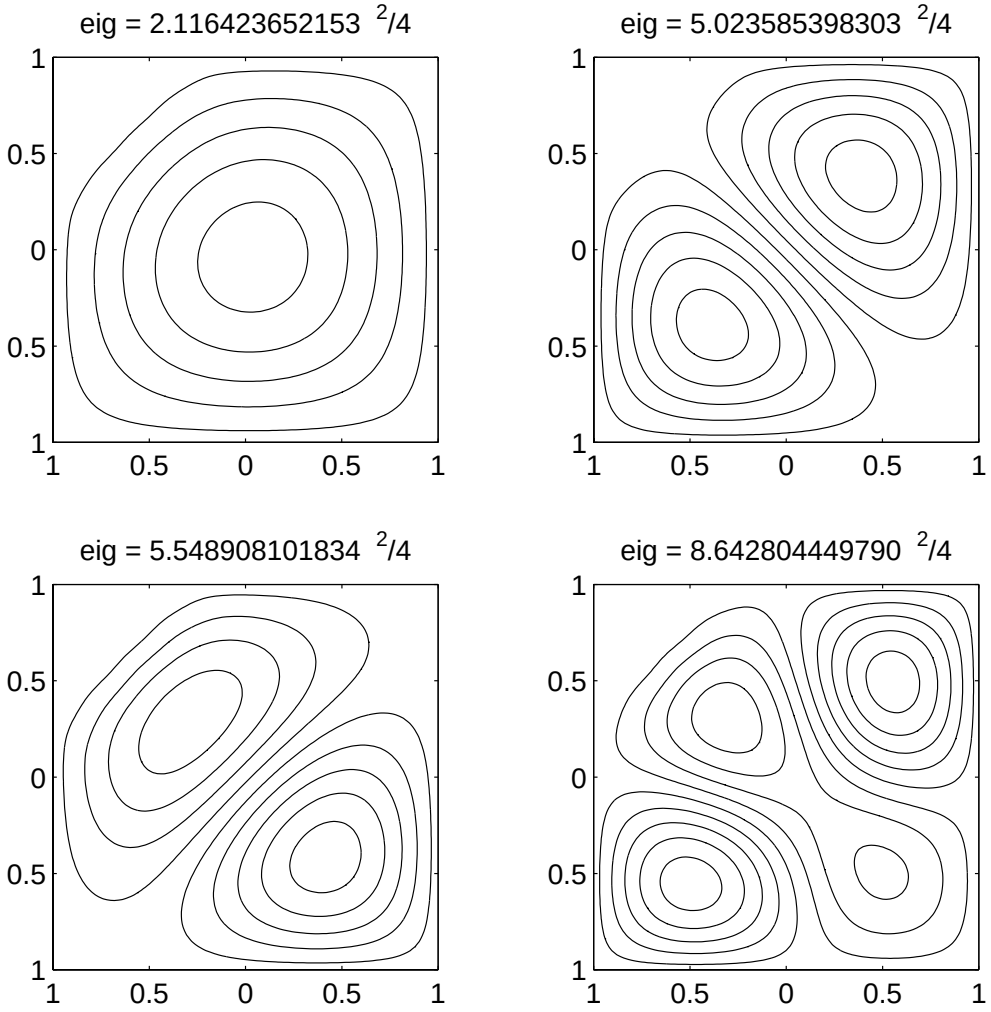

N = 24 eig = 501.3517186350
1
0.5
0
0.5
1
1
0.5
0
0.5
1

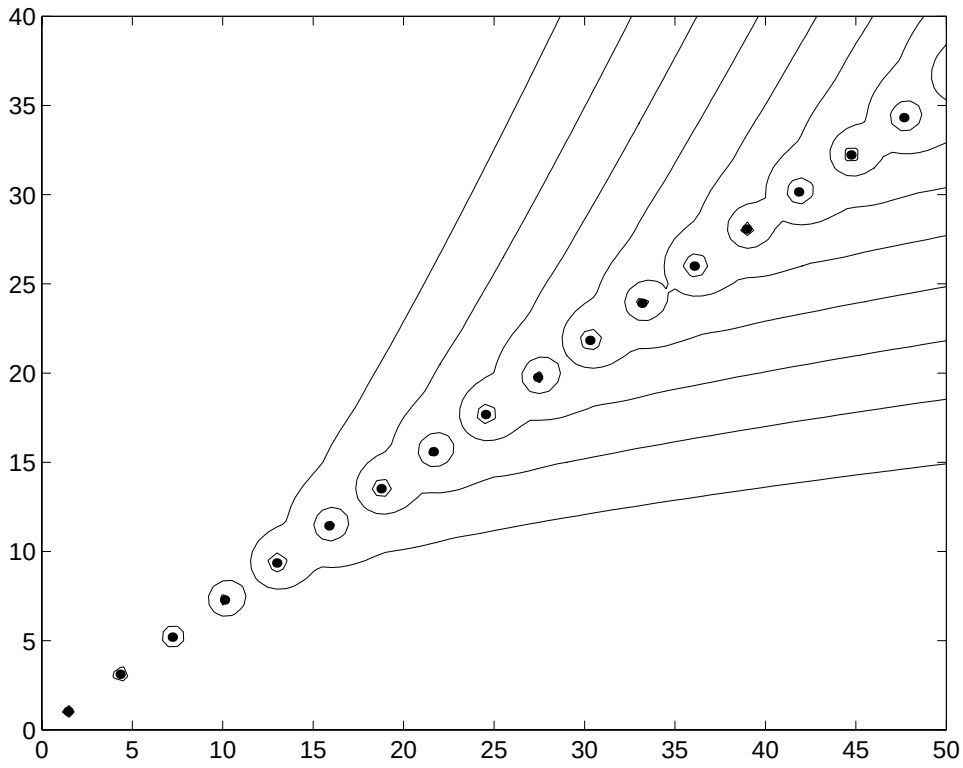

N = 36 eig = 501.3483797471
1
0.5
0
0.5
1
1
0.5
0
0.5
1


N = 48 eig = 501.3483797111
1
0.5
0
0.5
1
1
0.5
0
0.5
1

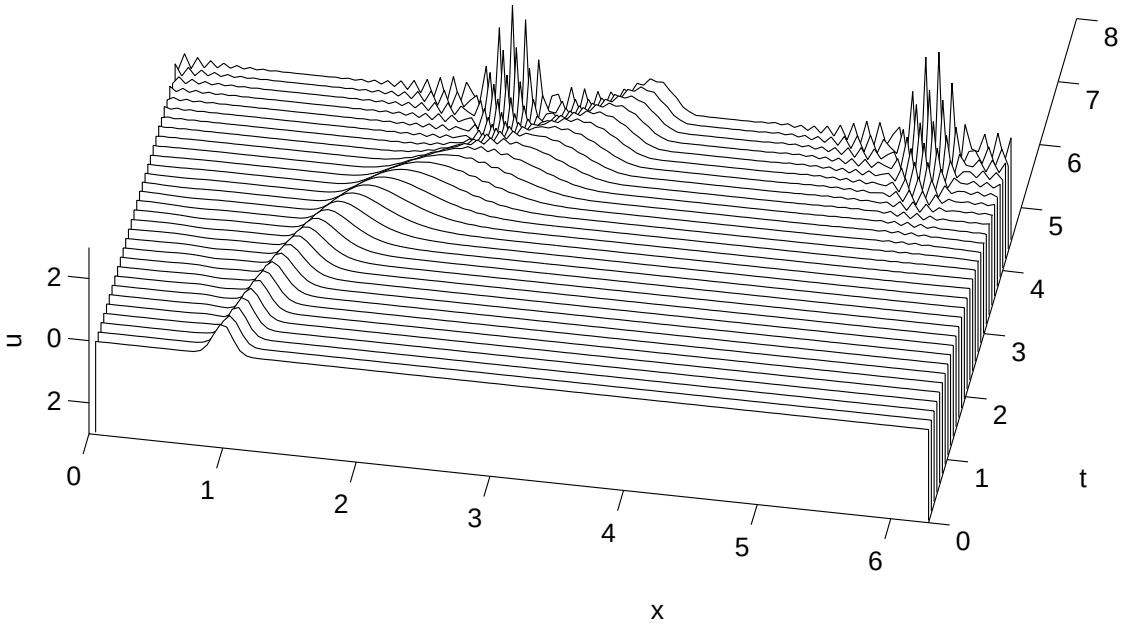


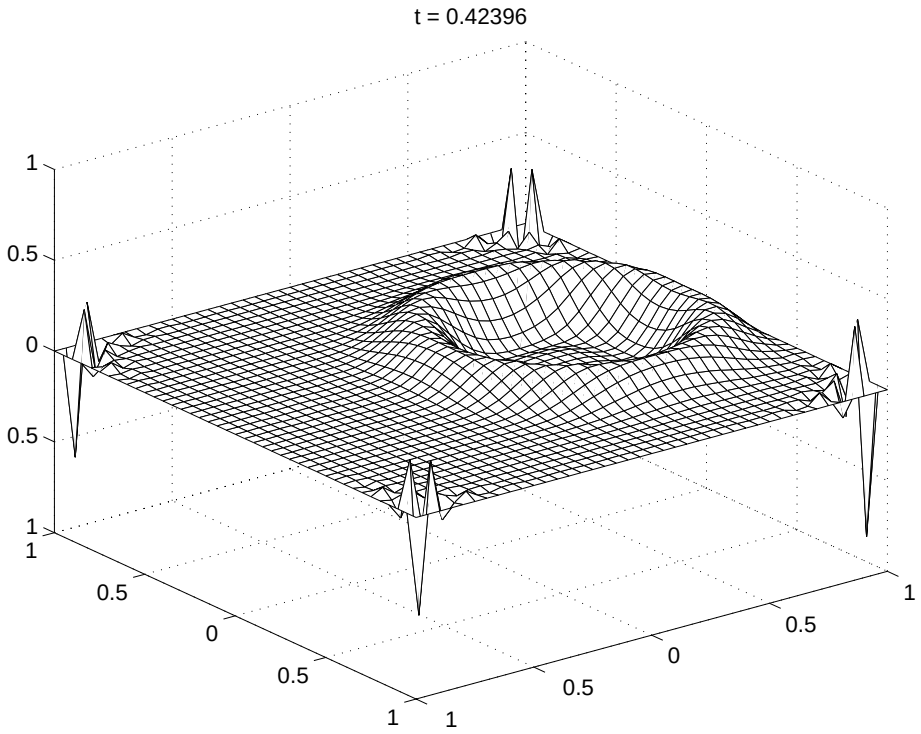

eig = 2.000000000000 2/4
eig = 5.000000000003 2/4
eig = 5.000000000004 2/4
eig = 8.000000000007 2/4
1
0.5
0
0.5
1

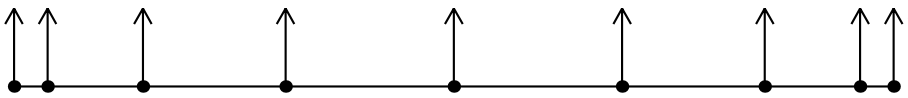

eig = 2.116423652153 2/4
eig = 5.023585398303 2/4
eig = 5.548908101834 2/4
eig = 8.642804449790 2/4
1
0.5
0
0.5
1

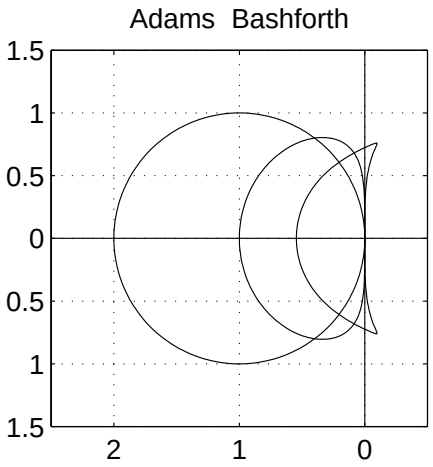

40
35
30
25
20
15
10
5
0
0
5
10
15
20
25
30
35
40
45
50

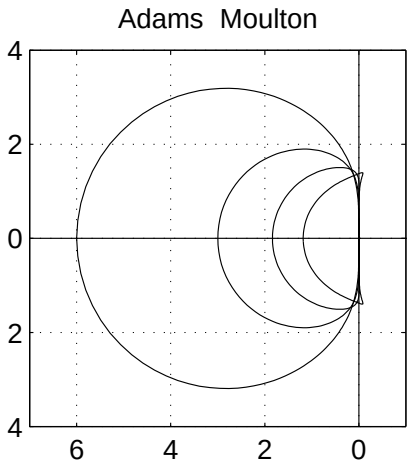
10. Time-Stepping and Stability Regions

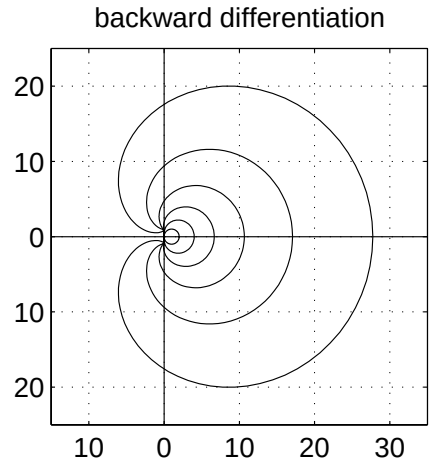

u
2
0
2
0
1
2
3
4
5
6
x
0
1
2
3
4
5
6
7
8
t

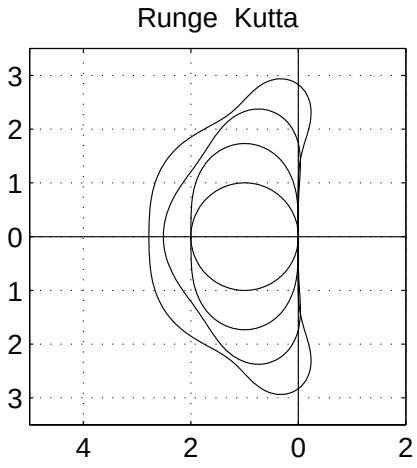

t = 0.42396
1
0.5
0
0.5
1
1
0.5
0
0.5
1
1
0.5
0
0.5
1

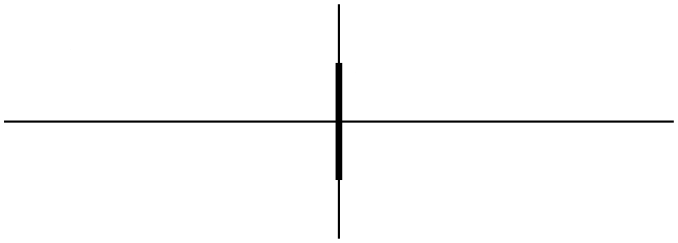


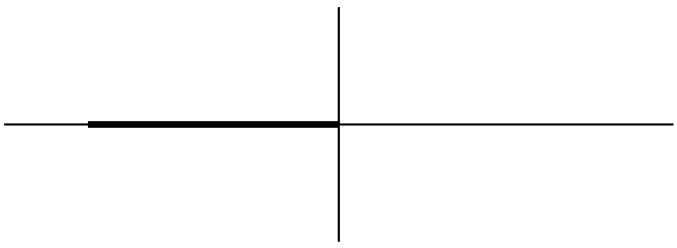

Adams Bashforth
1.5
1
0.5
0
0.5
1
1.5
2
1
0

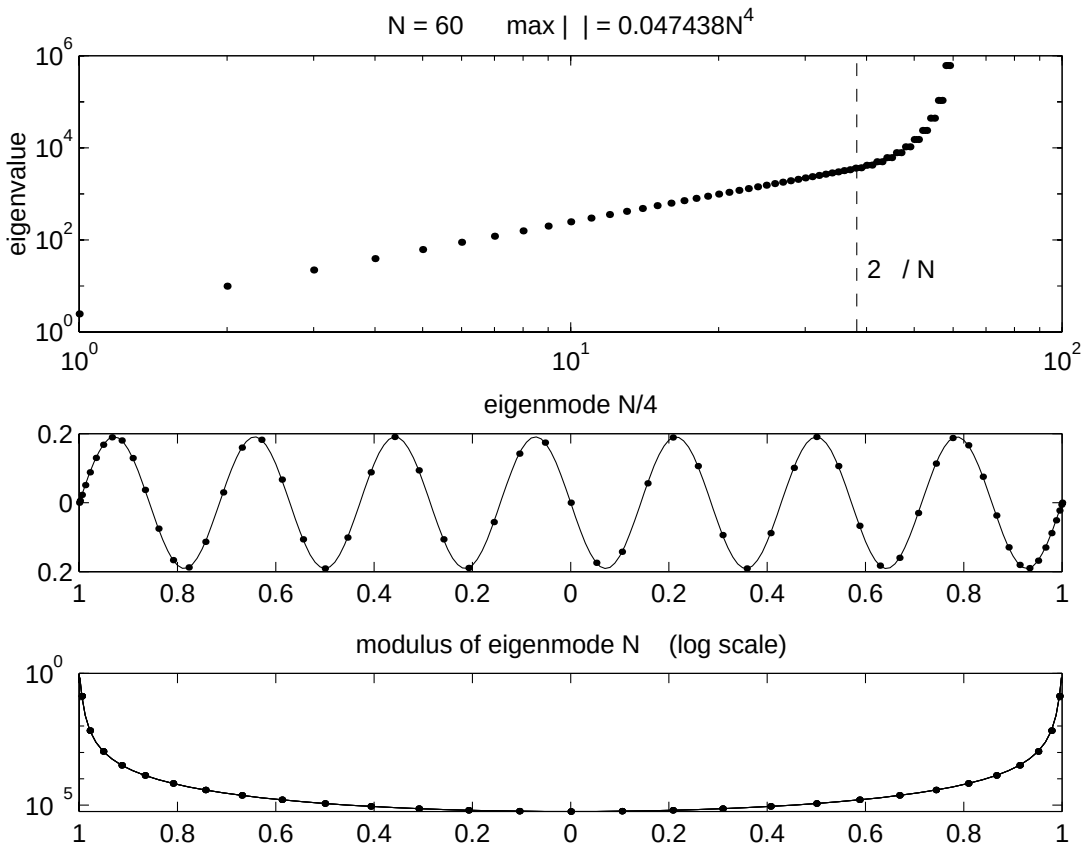

Adams Moulton
4
2
0
2
4
6
4
2
0

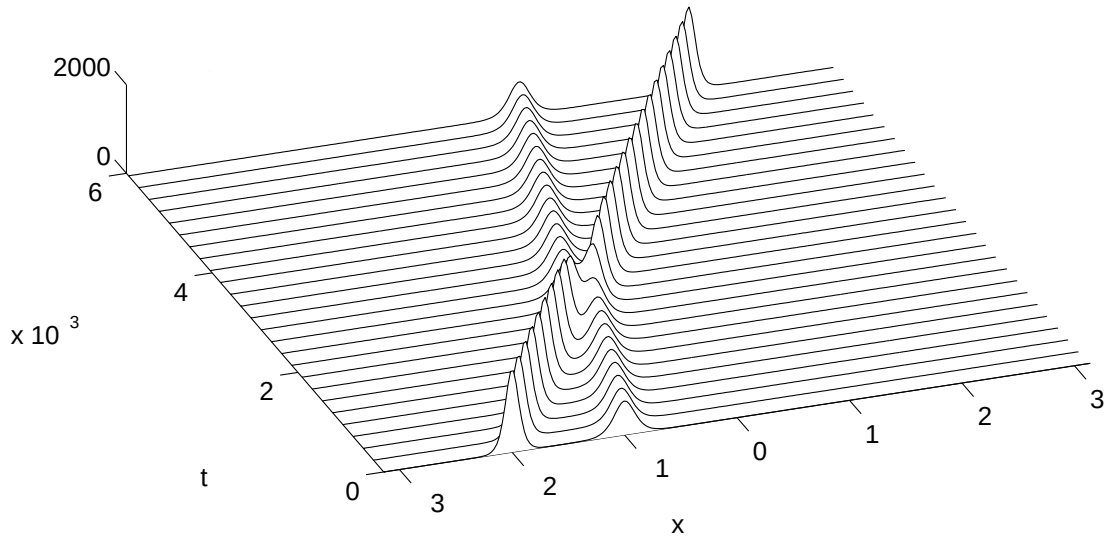

backward differentiation
20
10
0
10
20
10
0
10
20
30


Runge Kutta
3
2
1
0
1
2
3
4
2
0
2

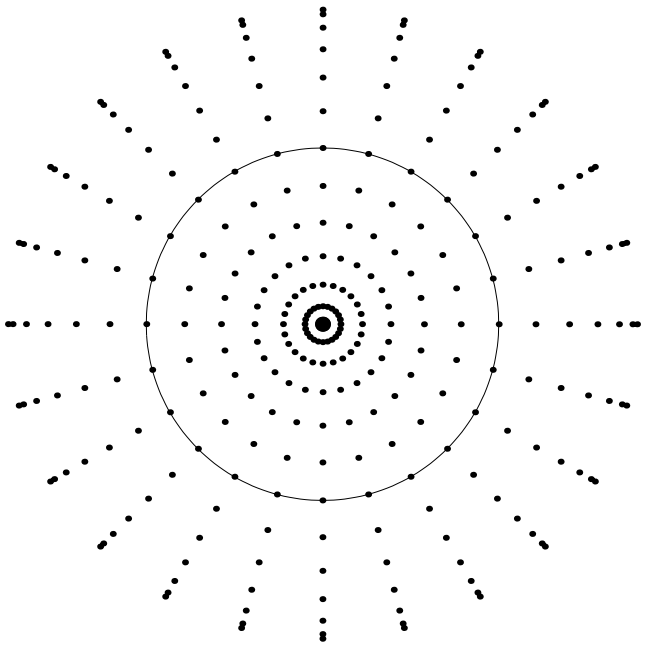


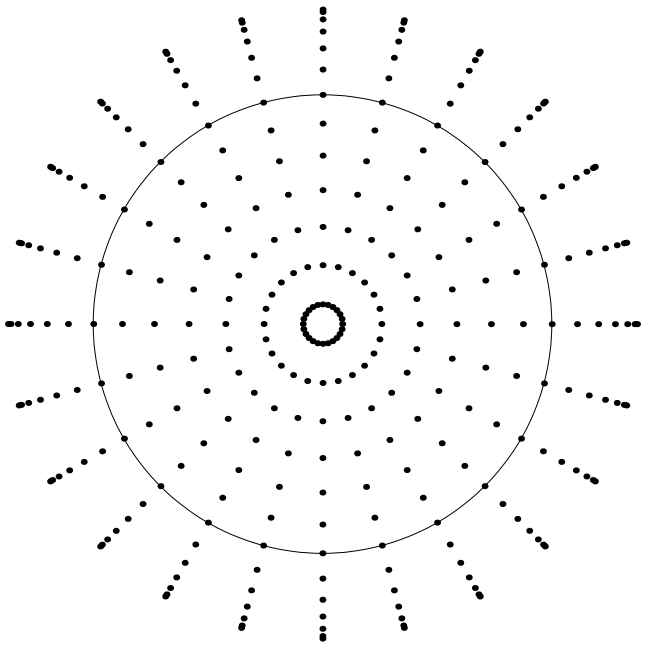



N = 60 max | | = 0.047438N^4
eigenvalue
10^6
10^4
10^2
10^0
10^0
10^1
10^2
2 / N
eigenmode N/4
0.2
0
0.2
1 0.8 0.6 0.4 0.2 0 0.2 0.4 0.6 0.8 1
modulus of eigenmode N (log scale)
10^0
10^5
1 0.8 0.6 0.4 0.2 0 0.2 0.4 0.6 0.8 1

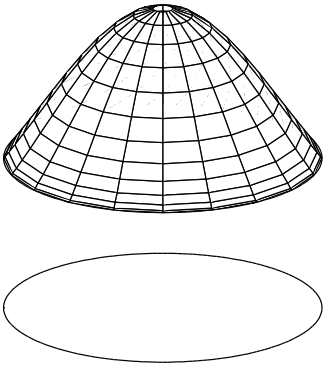

2000
0
6
4
x 10 3
2
t
0
3
2
1
0
1
2
3
x

11. Polar Coordinates

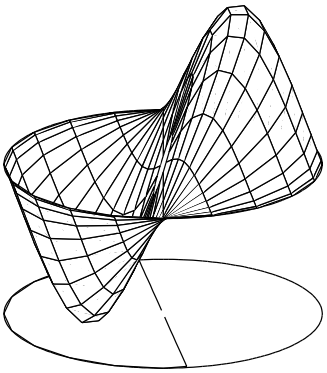




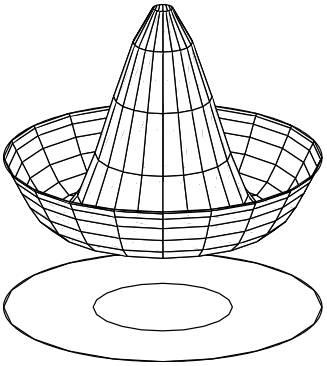
Mode 1
= 1.0000000000



Mode 2
= 1.5933405057



Mode 6
= 2.2954172674



Mode 9
= 2.9172954551